VECTOR SPACES

Vector space:-

A non-empty set V over the field $\mathbb{F}$ under the addition and scalar multiplication is said to be a vector space if it satisfy the following properties:-

(1) If $u, v \in V \Rightarrow u+v \in V$

(2) If $u, v, w \in V$, then $(u+v)+w = u+(v+w)$

(3) Existence of identity under addition

i.e. $u+0 = u = 0+u$

(4) Existence of inverse

$u+(-u) = 0 = (-u)+u$

(5) $u+v = v+u$

(6) If α is a scalar and $u \in V$, then $\alpha u \in V$.

(7) $\alpha(u+v) = \alpha u + \alpha v$, $\alpha \in \mathbb{F}$, $u, v \in V$

(8) $(\alpha+\beta)u = \alpha u + \beta u$, $\alpha, \beta \in \mathbb{F}$, $u \in V$

(9) $\alpha(\beta u) = (\alpha\beta)\cdot u = \beta\cdot(\alpha u)$, $\alpha, \beta \in \mathbb{F}$, $u \in V$

(10) $1 \cdot u = u$, $u \in V$.

Ex:-1 Set of real numbers,

$V = \mathbb{R}$, $\mathbb{F} = \mathbb{R}$.

Ex:-2 $\mathbb{R}(\mathbb{C})$ is not a vector space.

$u \in \mathbb{R}$

But $\alpha u \notin \mathbb{R}$, $\alpha \in \mathbb{C}$