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Direct sum of two subspaces :-

Let U and W are two subspaces of a vectorspace V then the direct sum of U and W is denoted by $U \oplus W$ and each element of this set is uniquely expressible.

$v = u + w$, $u \in U$ and $w \in W$.

Ex :- Let $V = \mathbb{R}^3$

$U = xy$ - plane

$W = yz$ - plane

$U \cap W = y$-axis $\neq \{0\}$

So $U \oplus W$ is not defined.

Ex :- $U = x$ - axis $\quad V = \mathbb{R}^2$

$W = y$ - axis

$\boxed{U \cap W = \{0\}}$

So we can define $U \oplus W$.

Theorem :-

Let U and W be two subspaces of V and $Z = U + W$.

Then $Z = U \oplus W$ iff the following conditions satisfied.

Any vector $z \in Z$ can be expressed uniquely as the sum

$z = u + w$, $u \in U$, $w \in W$