Proof:-

N.P:- Let $Z = U \oplus W$

Let z is not uniquely expressible as the sum of the elements of U and W

$z = u_1 + w_1$ and $z = u_2 + w_2$

$\Rightarrow u_1 + w_1 = u_2 + w_2$

$\Rightarrow u_1 - u_2 = w_2 - w_1$, $u_1 - u_2 \in U$
$w_2 - w_1 \in W$

$U \cap W = \{0\}$

$\Rightarrow u_1 - u_2 = 0 = w_2 - w_1$

$\Rightarrow u_1 = u_2$
$w_1 = w_2$.

which is a contradiction.

So z is uniquely expressible as the sum of the elements of U and W.

which proves the N.P.

S.P:- Suppose that z is uniquely expressible as the sum

$z = u + w$, $u \in U$, $w \in W$

Let $U \cap W \neq \{0\}$

let $v \in U \cap W$

$\Rightarrow v \in U \wedge v \in W$

Since $Z = U + W$

$v \in Z$

$v = 0 + v$, $0 \in U$, $v \in W$

$= v + 0$, $v \in U$, $0 \in W$