* Find a homogeneous system whose solution set W is spanned by $\{(1,-2,0,3), (1,-1,-1,4), (1,0,-2,5)\}$

A:-

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & x \\ -2 & -1 & 0 & y \\ 0 & -1 & -2 & z \\ 3 & 4 & 5 & t \end{array}\right) \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 + 2R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & x \\ 0 & 1 & 2 & y+2x \\ 0 & -1 & -2 & z \\ 0 & 1 & 2 & t-3x \end{array}\right)$$

$$\xrightarrow[R_4 \to R_4 - R_2]{R_3 \to R_3 + R_2} \underbrace{\left(\begin{array}{ccc|c} 1 & 1 & 1 & x \\ 0 & 1 & 2 & y+2x \\ 0 & 0 & 0 & z+y+2x \\ 0 & 1 & 2 & t-5x-y \end{array}\right)}_{M}$$

$A$

$$\text{rank}(A) = \text{rank}(M)$$

$$2x+y+z=0$$
$$t-5x-y=0$$

Theorem:-

Let U, be subspace of a vectorspace V which is finite dimensional then $\dim(U) \leq \dim(V)$.

* $\boxed{\dim(U+W) = \dim(U) + \dim(W) - \dim(U \cap W)}$.

P Kalika Maths