

$$(i) \underline{[B] \subseteq U+W}$$

Let $x \in [B]$

$$\Rightarrow x = \alpha_1 v_1 + \dots + \alpha_r v_r + \beta_{r+1} u_{r+1} + \dots + \beta_m u_m + \gamma_{r+1} w_{r+1} + \dots + \gamma_p w_p$$

$\in U+W$

$$\therefore [B] \subseteq U+W. \quad \text{--- (1)}$$

$$(ii) \underline{U+W \subseteq [B]}$$

Let $y \in U+W$

$$\Rightarrow y = \alpha_1 v_1 + \dots + \alpha_r v_r + \beta_{r+1} u_{r+1} + \dots + \beta_m u_m + \dots$$

$$+ \gamma_1 v_1 + \dots + \gamma_r v_r + d_{r+1} w_{r+1} + \dots + d_p w_p$$

$$= (\alpha_1 + \gamma_1) v_1 + \dots + (\alpha_r + \gamma_r) v_r + \beta_{r+1} u_{r+1} + \dots + \beta_m u_m$$

$$+ d_{r+1} w_{r+1} + \dots + d_p w_p$$

$$= \delta_1 v_1 + \dots + \delta_r v_r + \beta_{r+1} u_{r+1} + \dots + \beta_m u_m + d_{r+1} w_{r+1}$$

$$+ \dots + d_p w_p$$

$\in [B]$

$$\therefore U+W \subseteq [B] \quad \text{--- (2)}$$

From (1) and (2), we get $U+W = [B]$.