

(ii) $u = \alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_n u_n$

Let $\alpha_1 = 1, \alpha_2 = \dots = \alpha_n = 0$

$u = u_1$

$T(u_1) = v_1$

T

(iii) Let T is not unique

Let J another map $F \neq T$ such that

$F(u) = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$

Now, $F(u) - T(u) = (\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n) - (\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n)$

$\Rightarrow F(u) = T(u)$

A linear transformation T is completely determined by its values on the elements of a basis.

$(\alpha_1 u_1 + \dots + \alpha_n u_n) \xrightarrow{T} (\alpha_1 v_1 + \dots + \alpha_n v_n)$

$\alpha_1 (u_1) + \dots + \alpha_n (u_n) \xrightarrow{T} \alpha_1 (v_1) + \dots + \alpha_n (v_n)$

$T(\alpha_1 u_1 + \dots + \alpha_n u_n) = \alpha_1 T(u_1) + \dots + \alpha_n T(u_n)$

$(\alpha_1 u_1 + \dots + \alpha_n u_n) \xrightarrow{T} (\alpha_1 v_1 + \dots + \alpha_n v_n)$

$(\alpha_1 u_1 + \dots + \alpha_n u_n) \xrightarrow{T} (\alpha_1 v_1 + \dots + \alpha_n v_n)$

$\alpha_1 u_1 + \dots + \alpha_n u_n \xrightarrow{T} \alpha_1 v_1 + \dots + \alpha_n v_n$

$\alpha_1 u_1 + \dots + \alpha_n u_n \xrightarrow{T} \alpha_1 v_1 + \dots + \alpha_n v_n$

$(\alpha_1 u_1 + \dots + \alpha_n u_n) \xrightarrow{T} (\alpha_1 v_1 + \dots + \alpha_n v_n)$

$(\alpha_1 u_1 + \dots + \alpha_n u_n) \xrightarrow{T} (\alpha_1 v_1 + \dots + \alpha_n v_n)$

$(\alpha_1 u_1 + \dots + \alpha_n u_n) \xrightarrow{T} (\alpha_1 v_1 + \dots + \alpha_n v_n)$