$$a_0 + a_1x + a_2x^2 + a_3x^3 = \alpha(1+x) + \beta(2+x) + \gamma x^2$$

$$= (\alpha+2\beta) + (\alpha+\beta)x + \gamma x^2$$

$\Rightarrow$ $a_3 = 0$

$a_2 = \gamma$

$a_1 = \alpha+\beta$

$a_0 = \alpha+2\beta$

$\Rightarrow$ $\beta = a_0 - a_1$

$\Rightarrow$ $\alpha = a_1 - a_0 + a_1 = 2a_1 - a_0$

$$\therefore\ a_0 + a_1x + a_2x^2 + a_3x^3 = (2a_1 - a_0)(1+x) + (a_0 - a_1)(2+x) + a_2x^2$$

$$T(a_0 + a_1x + a_2x^2 + a_3x^3) = (2a_1 - a_0)(1+x) + (a_0 - a_1)(x + 3x^2) + a_2$$

Consider $\{1+x,\ 2+x,\ x^2,\ x^3\}$

The set is LI.

Assume $T(x^3) = 0$

$$\begin{vmatrix} 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1(1) - 1\begin{vmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 1 - 2 = -1 \neq 0$$

$\therefore$ The set is LI.