$\Rightarrow a_0 = -2a_3$

$\therefore U \cap W = \{-2a_3 + (-2a_2 - 3a_3)x + a_2x^2 + a_3x^3 \mid a_2, a_3 \in \mathbb{R}\}$

$= \{a_2(x^2 - 2x) + a_3(x^3 - 3x - 2) \mid a_2, a_3 \in \mathbb{R}\}$

$\dim(U \cap W) = 2$

Now, $U + W = \{a(x-2) + b(x^2-4) + c(x^3-8) + d + e(x^2-2x) + f(x^3-3x) \mid a, b, c, d, e, f \in \mathbb{R}\}$

$\dim(U+W) = \dim(U) + \dim(W) - \dim(U \cap W)$

$= 3 + 3 + 2$

$= 4$

$\therefore$ The set $\{(x-2), (x^2-4), (x^3-8), 1, (x^2-2x), (x^3-3x)\}$ is LD.

Now
$$\begin{matrix}1\\2\\3\\4\\5\\6\end{matrix}\begin{pmatrix}-2 & 1 & 0 & 0\\ -4 & 0 & 1 & 0\\ -8 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & -2 & 1 & 0\\ 0 & -3 & 0 & 1\end{pmatrix} \xrightarrow{R_4 \leftrightarrow R_1} \begin{matrix}4\\2\\3\\1\\5\\6\end{matrix}\begin{pmatrix}1 & 0 & 0 & 0\\ -4 & 0 & 1 & 0\\ -8 & 0 & 0 & 1\\ -2 & 1 & 0 & 0\\ 0 & -2 & 1 & 0\\ 0 & -3 & 0 & 1\end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_4}$$

$$\begin{matrix}4\\1\\3\\2\\5\\6\end{matrix}\begin{pmatrix}1 & 0 & 0 & 0\\ -2 & 1 & 0 & 0\\ -8 & 0 & 0 & 1\\ -4 & 0 & 1 & 0\\ 0 & -2 & 1 & 0\\ 0 & -3 & 0 & 1\end{pmatrix} \xrightarrow[R_3 \to R_3 + 8R_1]{R_2 \to R_2 + 2R_1,\ R_4 \to R_4 + 4R_1} \begin{matrix}4\\1\\3\\2\\5\\6\end{matrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & -2 & 1 & 0\\ 0 & -3 & 0 & 1\end{pmatrix}$$