and the angular velocity is given by

$$\dot{\psi} = \frac{\sqrt{GMa(1-e^2)}}{d^2}.$$

In equations (1) and (2), assume $\langle \bullet \rangle$ denotes an average in time over one orbit. For a function w of ψ this means that

$$\langle w \rangle = \frac{1}{T} \int_0^T w\big(\psi(t)\big)\,\mathrm{d}t = \frac{1}{T} \int_0^{2\pi} \frac{w(\psi)}{\dot{\psi}} \mathrm{d}\psi$$

where

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM}}$$

is the period of the orbit.

Assume that a and e do not change appreciably in one orbit, so that their time derivatives can be ignored in the averages.

Question 2 Note that $J_1 = J_2 = 0$ (i.e., the x and y components of $\mathbf{J}$ vanish). Show that

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}t} \right\rangle = -\frac{32G^4\mu^2M^3}{5c^5a^5} f(e) \tag{3}$$

where

$$f(e) = \frac{1 + \frac{73}{24}e^2 + \frac{37}{96}e^4}{(1-e^2)^{7/2}}$$

and that

$$\left\langle \frac{\mathrm{d}J_3}{\mathrm{d}t} \right\rangle = -\frac{32G^{7/2}\mu^2M^{5/2}}{5c^5a^{7/2}} g(e) \tag{4}$$

where

$$g(e) = \frac{1 + \frac{7}{8}e^2}{(1-e^2)^2}.$$

For Keplerian orbits we have

$$E = -G\frac{\mu M}{2a}$$

and

$$J_3 = \sqrt{G\mu^2 Ma(1-e^2)}$$

(J_3 is usually denoted L).

Question 3 For Keplerian orbits prove that

$$\left\langle \frac{\mathrm{d}a}{\mathrm{d}t} \right\rangle = -\frac{64G^3\mu M^2}{5c^5a^3} f(e),$$

and that

$$\left\langle \frac{\mathrm{d}e}{\mathrm{d}t} \right\rangle = -\frac{304G^3\mu M^2}{15c^5a^4} h(e)$$

where

$$h(e) = \frac{(1 + \frac{121}{304}e^2)e}{(1-e^2)^{5/2}}.$$

Under what conditions is it true that the time derivatives of a and e (as just calculated) can be ignored in the averages in (1) and (2) (as was assumed above in deriving (3) and (4))? (This is a consistency check.)

